

A comparison of the gear calculation process according to Swedish and American textbooks for higher education

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ABSTRACT: Gears of different types are used to transmit torque and rotation from one shaft to another. Gears are among the oldest machine elements. The shape of the gear tooth has been improved through the history and today's gear teeth are formed by the involute curve, hence the name involute gears. The involute shape makes the gears to transmit rotation between shafts at a constant rate and with very low friction losses. Different types and arrangements of gears are used in different situations, but only external spur gears, which transmit rotational speed between parallel shafts, are dealt with in this article. The reason for this selection is that spur gears are the most simple and are the base of other gears. Notably, gear calculation processes in Swedish textbooks (Swedish Standard) and American textbooks (AGMA standard) for higher education are presented in this article. Some test results have been calculated using different modules, gear teeth numbers and gear ratios. The results show big differences between Swedish and American process calculations regarding gear teeth bending stresses. These differences are so large that mistakes in calculations can be suspected, although the calculations have been checked many times by different people. There are also clear differences regarding gear tooth surface stresses.

INTRODUCTION

Gears, defined as toothed members that transmit torque and angular velocity from one shaft to another, are used in a wide variety of applications. Gears are among the oldest devices and human inventions. The ancient Chinese South-Pointing Chariot from 2600 BC contained gears in a complex design. Aristotle, in the 4th Century BC, wrote of gears as though they were commonplace. In the 15th Century, Leonardo da Vinci designed a multitude of devices incorporating many kinds of gears [1].

Early gears were most likely made crudely from wood and other easily worked materials, their teeth merely being pegs inserted in a disk or wheel (see Figure 1, right). It was not until the Industrial Revolution that machines demanded, and manufacturing techniques allowed, the creation of gears as they are known today with special shaped teeth formed or cut into a metal disk (see Figure 1, left) [2].

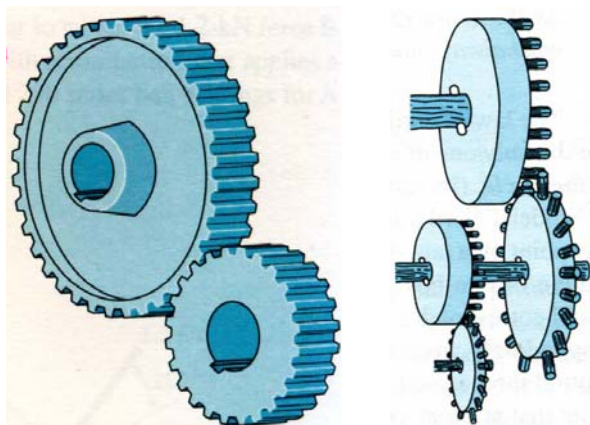


Figure 1: Modern gears (a spur gear) (left) and old gears made from wood [1].

Among the various means of mechanical power transmission (gears, chains and belts), gears are generally the most rugged and durable and their transmission efficiency can be as high as 98% [3]. Gears are more costly than chain and belt drives. The primary function of a gear is to transfer power from one shaft to another while maintaining a definite ratio between the velocities of the shaft rotations. The teeth of the driving gear mesh push on the driven gear teeth, exerting a force component perpendicular to the gear radius. Thus, torque is transmitted and, because the gear is rotating, power is transferred [4]. Gears are highly standardised as to tooth shape and size.

There are different types of gears used in different situations. For example, spur and helical gears are used to transmit rotary motion between parallel shafts while bevel gears are used for intersecting shafts. A third variant is the worm gear, which is used to transmit rotary motion between nonparallel and nonintersecting shafts [5]. Figure 2 shows diagrams of these three examples.

Spur and helical gears are cylindrical and quite identical except the helix angle in the last group. The helix angle makes the helical gears stronger and more silent than the spur gears; therefore, they are used in automobiles and other designs with noise reduction requirements.

Spur gears are the simplest of all gear types and represent the basic of the gear calculation theories. In this article, the author deals only with *external* spur gears because this group is representative for all the gears. Besides that, to make comparisons between all the gear types would require a lot of space and a large amount of time.

The idea of this comparison is to examine if there are any differences in the calculation results derived from the American and Swedish calculation processes for spur gears.

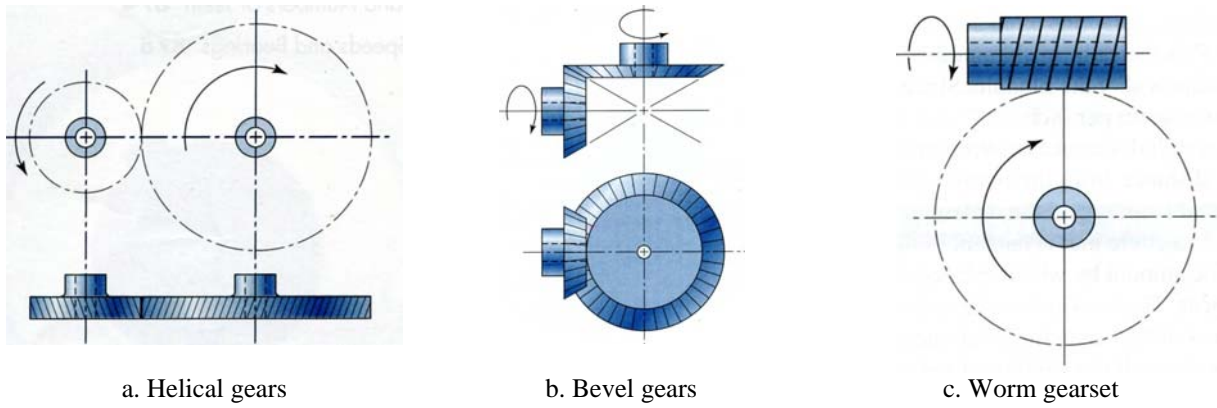


Figure 2: Some different types of gears [5].

Gear Geometry

The basic requirement of gear tooth geometry is the condition of angular velocity ratios that are exactly constant. This requirement is satisfied by designing the gear tooth shape utilising involute curves (see Figure 3a). The pitch circles of two mating gears can be compared with two cylinders pressed together (see Figure 3b).

The terminology of spur-gear teeth is illustrated in Figure 4. The pitch circle is a theoretical circle upon which all calculations are usually based; its diameter is the pitch diameter [5]. A pinion is the smaller of two gears. The larger is often called the gear.

The circular pitch p is the distance, measured on the pitch circle, from a point on one tooth to a corresponding point on an adjacent tooth. The module m is the ratio of the pitch diameter to the number of teeth. The module is the index of tooth size in the Swedish Standard, which is based on ISO standard and SI units. The diametral pitch P is the ratio of the number of teeth on the gear to the pitch diameter. The diametral pitch is only used in the American standard or American Gear Manufacturers Association (AGMA) Standard and is expressed as teeth per inch. Table 1 shows some standard modules used in the Swedish and American Standards. The rest of the gear tooth geometry is shown in Figure 4. The tooth system is a standard that specifies the relationships involving addendum, dedendum, working depth, tooth thickness and pressure angle. In order

to make a comparison possible, only the SI-units (metric system) is used.

The standard of the gear geometries of the USA and Sweden is quite similar except a few differences. One of these differences is the pressure angle ϕ (or α in Swedish Standard) (see Figure 5). This angle is determined to be 20° in the Swedish Standard, but it can be 20° , 22.5° or 25° in the American Standard. The gear teeth form will, of course, change depending on the size of pressure angle.

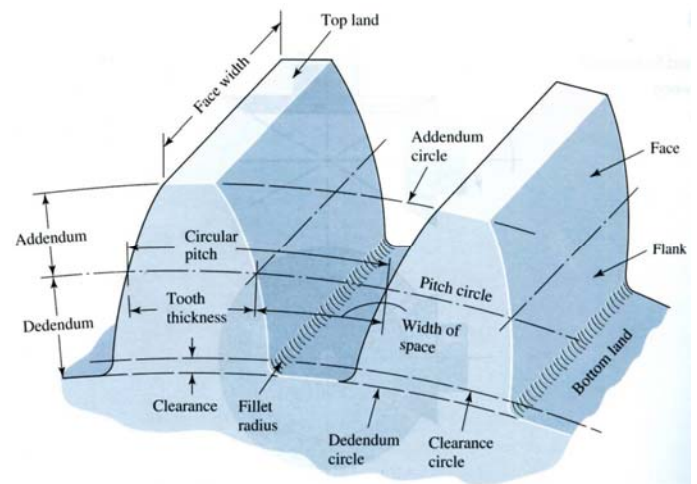
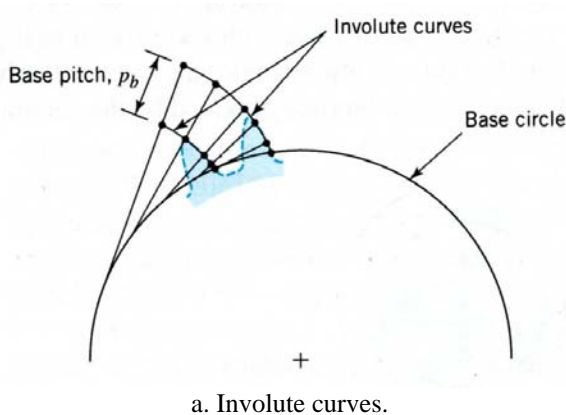
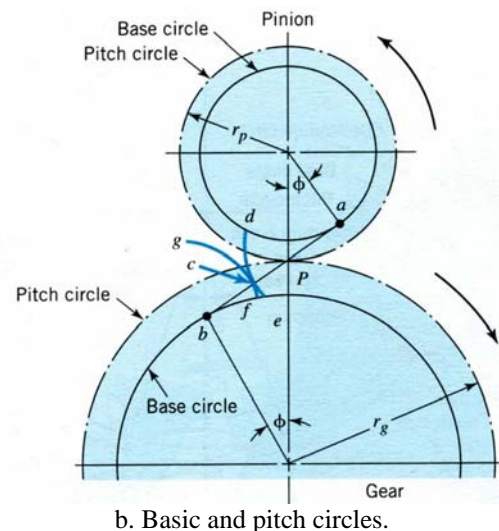


Figure 4: Nomenclature of spur-gear teeth [5].



a. Involute curves.



b. Basic and pitch circles.

Figure 3: Showing the involute curves of teeth shape and circles in two mating gears [1].

Table 1: Some modules used in the standards of Sweden and the USA [5].

Modules m [mm]												
Preferred	1	1.25	1.5	2	2.5	3	4	5	6	8	10	12
Next choice	1.125	1.375	1.75	2.25	2.75	3.5	4.5	5.5	7	9	11	14

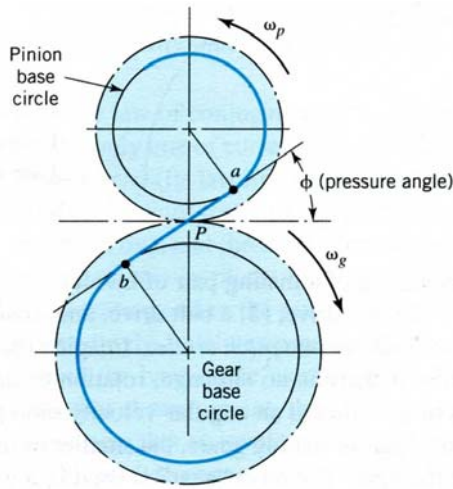


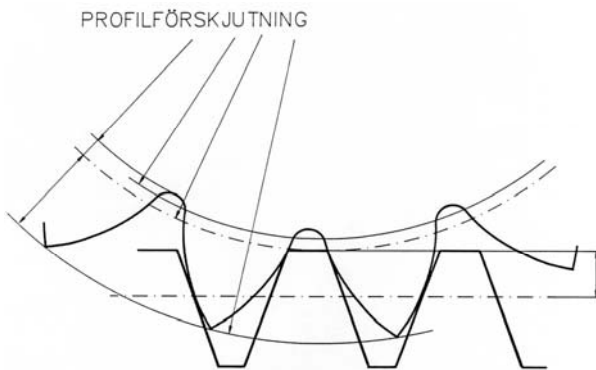
Figure 5: The pressure angle [1].

Swedish Standard. Since not one of the main American reference books mentions this method, the author assumes that it is not used in the American Standard.

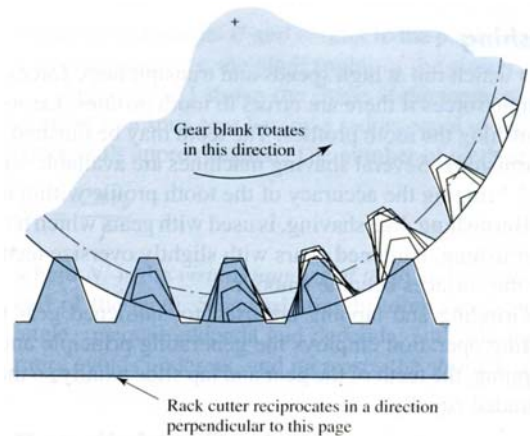
The addendum modification is carried out by placing the gear cutting tool closer (negative addendum modification) or away from the gearwheel material (positive addendum modification). The advantage of this method is to change the shape of the gear teeth, for example to increase the tooth bending strength, without changing the tool (see Figure 6). In Figure 6a, it can be seen how the positive addendum modification increases the tooth's thickness on the bottom and decreases it on the top of the tooth. The bending strength will, in this case, be better compared with non-addendum modified gear. Figure 6b shows a similar picture but without any addendum modifications.

Table 2 shows the names and designations of some gear geometries from the standards of both countries. These geometries are calculated before the calculation process of the stresses.

Another difference between the two standards that affects the teeth shape is the application of addendum modifications in the



a. The cutting tool moved away from the gear material, positive addendum modification [6].



b. Cutting tool shaping the gear without addendum modification [5].

Figure 6: How the cutting tool (rack cutter) shapes the teeth.

Table 2: Some teeth geometry nomenclature in the both standards [5][7].

Name	Swedish Designation (ISO)	American Designation (AGMA)
Module	m in [mm]	m in [mm]
Circular pitch	$p = m \cdot \pi$, see figure 4	p in [mm], see figure 4
Diametral pitch	-----	P [teeth per inch]
Number of teeth	z	N
Addendum	$h_a = m$	a = m (full depth) a = 0.8m (stub)
Dedendum	$h_f = 1.25m$	b = 1.25m to 1.35m (full depth) b = m (stub)
Pressure angle	$\alpha = 20^\circ$	$\phi = 20^\circ$, $\phi = 22.5^\circ$ or $\phi = 25^\circ$
Whole depth	$h = h_a + h_f$	$h_t = a + b$
Pitch diameter	$d = mz$	$d = mz$
Addendum diameter	$d_a = d + 2h_a$	$d_a = d + 2a$
Dedendum diameter	$d_f = d - 2h_f$	
Base diameter	$d_b = d \cdot \cos \alpha$	$d_b = d \cdot \cos \phi$

Gear Tooth Stresses

When two mating gears are pressed against each other, stress occurs at two locations: the first one is at the point of contact with the mating gear, where gear force F is acting. The second is in the fillet at the base of the tooth (see Figure 7). The tangential component F_t of the gear force multiplied by the pitch radius gives the bending moment that load the gear teeth root. The gear force F presses the mating gears together and this is a special situation of two parallel cylinders in contact. Figure 8 illustrates the gear loads (forces) [1].

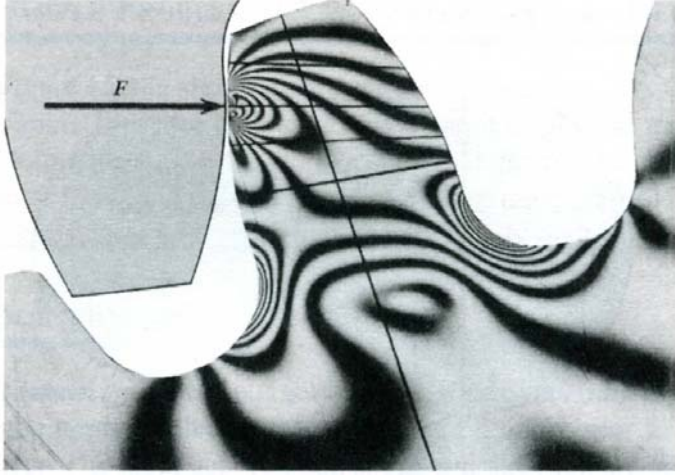


Figure 7: Gear tooth stresses [1].

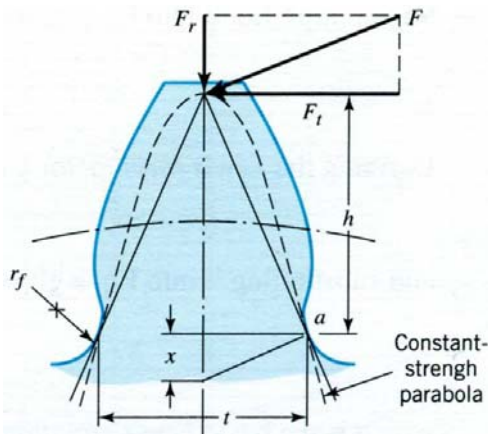


Figure 8: Gear loads (forces) [1].

The Gear Stresses Calculation Process in Swedish Textbooks

As mentioned before, the gear calculations process in the Swedish textbooks is taken from the Swedish standard, which is quite similar to the ISO standard [8]. The stress calculation equations are the same for spur and helical gears, except for some factors that are different. The equations and the factors are as detailed below (the Swedish equations are taken from the Swedish Standard SS 1871 [9]).

The calculation load (force) F_{ber} must be calculated before calculating the stresses [9].

$$\text{Calculation load: } F_{ber} = \frac{2T_1}{d_1} \times K_1 \times K_v \quad (1)$$

where:

T_1 = torque in the pinion;
 d_1 = pinions pitch diameter;
 K_1 = load factor. K_1 is from Table 3. Uniform motion is assumed in this case.

Gear tooth bending stress:

$$\sigma_F = Y_F \times Y_\beta \times Y_\varepsilon \times \frac{F_{ber} \times K_{F\alpha} \times K_{F\beta}}{b \times m_n} \quad (2)$$

where:

Y_F = bending stress concentration factor. This factor is estimated from Figure 9. The gear tooth root fillet radius is 0.38m;
 Y_β = helix angle factor. $Y_\beta = 1$ for spur gears;
 Y_ε = contact ratio factor. $Y_\varepsilon = 1/\varepsilon_\alpha$ where ε_α = contact ratio;
 $K_{F\alpha}$ = load distribution factor. Normally $K_{F\alpha} = 1$;
 $K_{F\beta}$ = load propagation factor. $K_{F\beta} = 1$ for spur gears;
 b = gear face width. In this case, it is $b = 10m$;
 m_n = module on the normal plane. For spur gear, this is $m_n = m$.

Gear tooth surface stress:

$$\sigma_H = Z_H \times Z_M \times Z_\varepsilon \sqrt{\frac{F_{ber} \times K_{H\alpha} \times K_{H\beta} (u-1)}{b \times d_1 \times u}} \quad (3)$$

where:

Z_H = stress concentration factor for surface fatigue at rolling point. For spur gear:

$$Z_H = \sqrt{\frac{2}{\sin 2\alpha}}; \text{ where } \alpha = \text{pressure angle} = 20^\circ;$$

$Z_m = \sqrt{0,35E}$ for metals in the booth gears where E = Young's modulus;

$$Z_\varepsilon = \sqrt{\frac{4 - \varepsilon_\alpha}{3}} \text{ for spur gears. } \varepsilon_\alpha = \text{contact ratio};$$

$K_{H\alpha}$ = load distribution factor. Normally $K_{H\alpha} = 1$;

$K_{H\beta}$ = load propagation factor. $K_{H\beta} = 1$ for spur gears;
 u = gear ratio. $u = n_1/n_2$. n_1 = pinions rotational speed and n_2 = gears rotational speed;

b = gear face width. In this case, $b = 10m$. m = module;

d_1 = pinions pitch diameter.

Table 3: Load factor K_1 [9].

Motion of the Source of Power	Motion of the Driven Machinery		
	Uniform	Moderate Shock	Heavy Shock
Uniform	1.00	1.25	≥ 1.75
Light shock	1.25	1.50	≥ 2.00
Medium shock	1.50	1.75	≥ 2.25

Fillet (root)

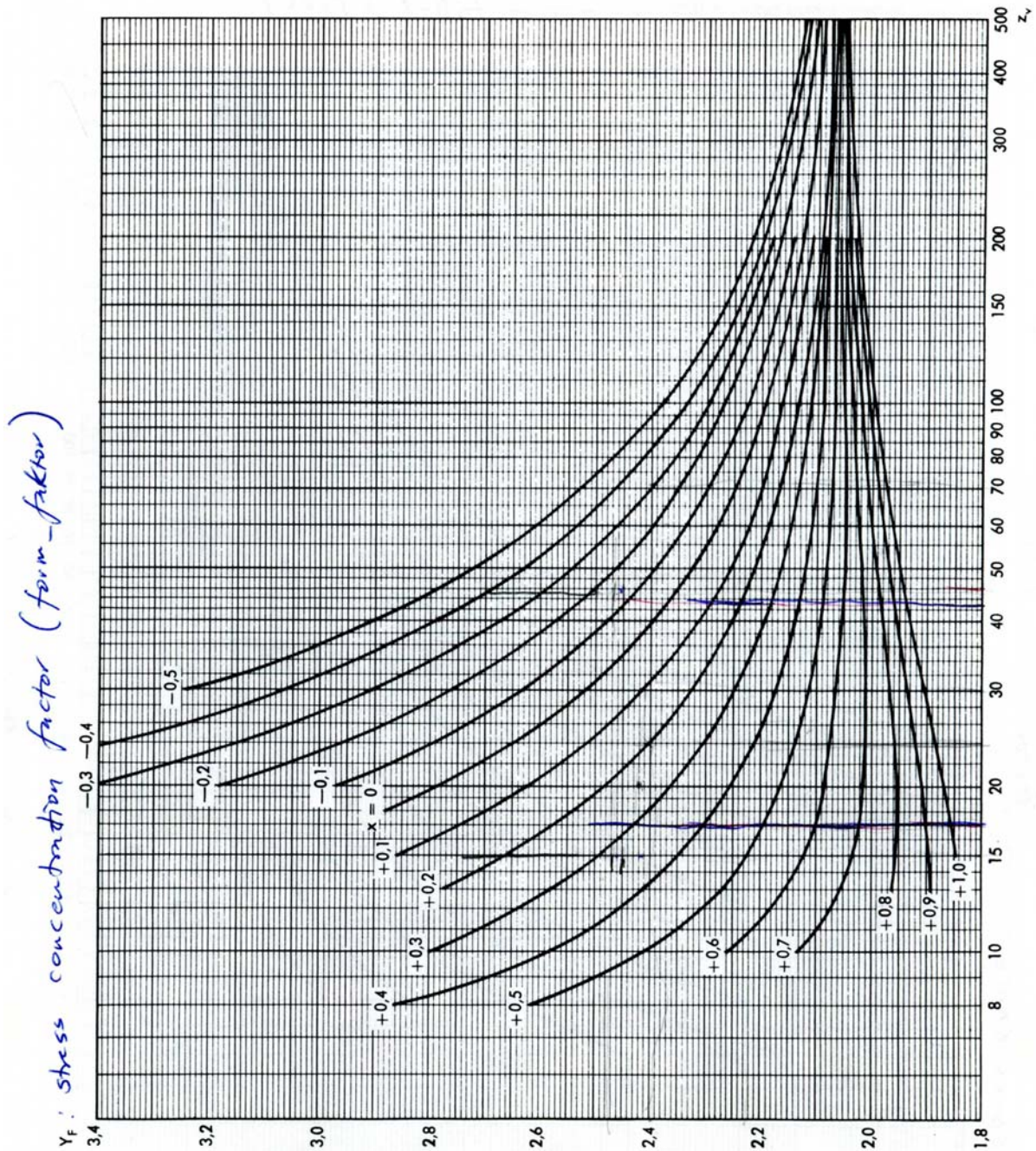


Figure 9: Bending stress concentration factor [9]. The apparent teeth number $z_v =$ teeth number z for spur gears.

The Gear Stresses Calculation Process in American Textbooks

The calculation equations, tables and graph in this section have been taken from Juvinal and Marshek [1]. The gear tooth stresses are calculated with following equations:

Gear tooth bending stress:
$$\sigma = \frac{F_t P}{b J} K_v K_o K_m \quad (4)$$

where:

F_t = Tangential component of the gear load F (see Figure 8);

$F_t = \frac{2T}{d}$ where T = torque and d = pitch diameter

(one pinion or gear);

P = diametral pitch, $P = 25.4/m$;
 b = gear teeth face width. In this comparison, the face width is decided to be 10m;
 J = spur gear geometry factor from Figure 10. Only 20° full-depth teeth gears are tackled in this article;
 K_v = velocity of dynamic factor, In this case, precision, shaved and ground spur gears are used. Therefore:

$$K_v = \frac{50 + \sqrt{V}}{50} \quad (5)$$

where V = pitch line velocity [feet/minute]
 K_o = overload factor. K_o is estimated from Table 4. Uniform motion is assumed in this case;
 K_m = mounting factor. This is estimated from Table 5. In this case, less rigid mountings, less accurate gears and contact across the full face are assumed.

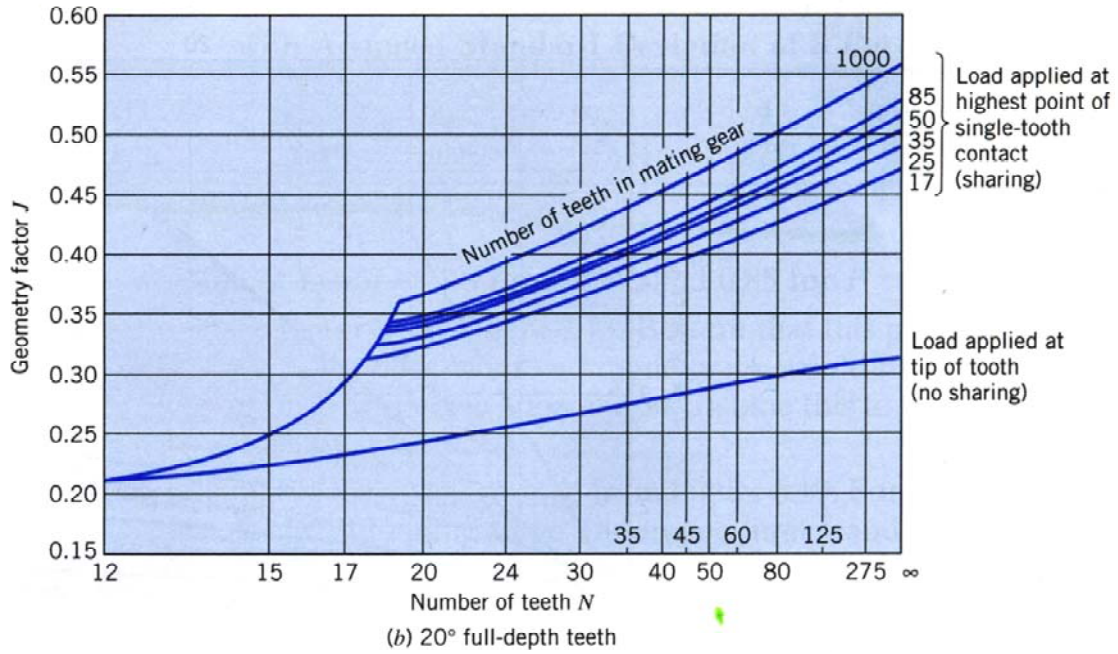


Figure 10: Geometry factor [1].

Table 4: Overload correction factor K_o [1].

Source of Power	Driven Machinery		
	Uniform	Moderate Shock	Heavy Shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Table 5: Mounting correction factor K_m [1].

Characteristics of Support	Face Width (inches)			
	0 to 2	6	9	16 up
Accurate mountings, small bearing clearances, minimum deflection, precision gears	1.3	1.4	1.5	1.8
Less rigid mountings, less accurate gears, contact across the full face	1.6	1.7	1.8	2.2
Accuracy and mounting such that less than full-face contact exists	Over 2.2			

The gear tooth surface stress:

$$\sigma_H = C_p \sqrt{\frac{F_t}{bd_p I} K_v K_o K_m} \quad (6)$$

where:

C_p = Elastic coefficient and can be selected from Table 6. Only steel gears are calculated in this paper.

$$I = \text{Geometry factor. } I = \frac{\sin \phi \cos \phi}{2} \frac{R}{R+1};$$

where:

ϕ = pressure angle = 20°

R is the ratio of gear and pinion diameters, $R = \frac{d_g}{d_p}$.

Table 6: Values of Elastic Coefficient C_p for Spur Gears, in \sqrt{MPa} .

Pinion Material ($\nu = 0.30$ in All Cases)	Gear Material			
	Steel	Cast Iron	Aluminium Bronze	Tin Bronze
Steel, $E = 207$ GPa	191	166	162	158
Cast iron, $E = 131$ GPa	166	149	149	145
Aluminium bronze, $E = 121$ GPa	162	149	145	141
Tin bronze, $E = 110$ GPa	158	145	141	137

Comparison and Result

A series of tests with different data are accomplished in order to compare the result of both calculation processes. The tests included different modules, different teeth numbers and different gear ratios for both standards. The modules ranged from 1 to 6 mm, the gear teeth numbers were from 15 to 120 teeth, and the gear ratio from 2 to 4. These tests have been used to calculate the gear tooth bending stresses and the gear tooth surface stresses using the both standards.

The results of the gear tooth bending stresses, which is given in MPa, are shown in Table 7. Also the ratio between the stresses calculated by the Swedish and the American processes are also shown in Table 7. The mean ratio of the result shows that the gear bending stresses calculation by the Swedish process make up only 27% of the stresses calculated by the American process.

The results of the gear tooth surface stresses, which is given in MPa, are shown in Table 8. Also the ratio between the stresses calculated by the Swedish and the American processes are likewise shown in Table 8. The mean ratio of the result shows that the gear surface stresses calculation by the Swedish process make up 75% of the stresses calculated by the American process.

CONCLUSION

The result of gear tooth bending stress indicates that the stress level is only 27% if it is calculated by the Swedish method compared with the American process. Some differences between the results of both calculation processes was expected, because some of the factors used in the equation are quite different from each other, but it was not expected that the difference would be so big. This means that an American gear should be about four times bigger than a Swedish one of the

same size and subjected to the same load with respect to gear teeth bending stress. Although the calculation has been checked several times by different people, and the calculations of the American process has been accomplished by both the SI units and British unit systems, it seems that some mistake had happened while using the American calculation method. Hopefully, someone will check this result and provide feedback.

Regarding gear tooth surface stresses, the result is much better. The result from the Swedish calculation process is 75% of the result from the American process. In general, the American calculation process gives more volume and, of course, a higher safety factor when compared with the European method. This may be because of the huge amount of damage incurred if the products fail and cause problems.

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Table 7: The result of the gear tooth bending stresses given in MPa, along with the ratio between the result of the Swedish and the American calculation processes.

	m [mm]	1	2	4	5	6	Ratio
Swedish	$N_1 = 30$	429					0.284
American	$N_2 = 60$	1,511					
Swedish	$N_1=30$	421	314				0.264
American	$N_2 = 90$	1,511	1,258				
Swedish	$N_1=30$	418	311				0.264
American	$N_2 = 120$	1,511	1,234				
Swedish	$N_1=25$		395				0.269
American	$N_2 = 50$		1,470				
Swedish	$N_1=20$			297			0.263
American	$N_2 = 40$			1,130			
Swedish	$N_1 = 20$			291		47	0.247
American	$N_2=60$			1,130		198	
Swedish	$N_1=20$			287			0.254
American	$N_1=80$			1,130			
Swedish	$N_1=15$				233	70	0.267
American	$N_2 = 30$				840	273	
Swedish	$N_1 = 15$				227		0.292
American	$N_2=45$				777		
Swedish	$N_1=15$				244		0.314
American	$N_1=60$				777		
Swedish	$N_1=25$					36	0.245
American	$N_1=100$					147	
Mean Ratio							0.270

Table 8: The result of the gear tooth surface stresses given in MPa, along with the ratio between the result of the Swedish and the American calculation processes.

	m [mm]	1	2	4	5	6	Ratio
<i>Swedish</i>	$N_1 = 30$	<i>1,563</i>					0.783
American	$N_2 = 60$	1,997					
<i>Swedish</i>	$N_1 = 30$	<i>1,465</i>	<i>1,263</i>				0.757
American	$N_2 = 90$	1,881	1,717				
<i>Swedish</i>	$N_1 = 30$	<i>1,413</i>	<i>1,219</i>				0.744
American	$N_2 = 120$	1,817	1,717				
<i>Swedish</i>	$N_1 = 25$		<i>1,603</i>				0.757
American	$N_2 = 50$		2,118				
<i>Swedish</i>	$N_1 = 20$			<i>1,498</i>			0.751
American	$N_2 = 40$			1,995			
<i>Swedish</i>	$N_1 = 20$			<i>1,402</i>		<i>564</i>	0.732
American	$N_2 = 60$			1,878		787	
<i>Swedish</i>	$N_1 = 20$			<i>1,351</i>			0.744
American	$N_1 = 80$			1,814			
<i>Swedish</i>	$N_1 = 15$				<i>1,440</i>	<i>789</i>	0.776
American	$N_2 = 30$				1,845	1,022	
<i>Swedish</i>	$N_1 = 15$				<i>1,346</i>		0.774
American	$N_2 = 45$				1,738		
<i>Swedish</i>	$N_1 = 15$				<i>1,297</i>		0.772
American	$N_1 = 60$				1,679		
<i>Swedish</i>	$N_1 = 25$					<i>443</i>	0.709
American	$N_1 = 100$					625	
Mean Ratio							0.750